

Exam. Code : 103201  
Subject Code: 1025

B.A./B.Sc. 1<sup>st</sup> Semester  
MATHEMATICS  
Paper—I (Algebra)

Time Allowed—3 Hours] [Maximum Marks—50

**Note** :— Attempt five questions in all, selecting at least one question from each Section. The fifth question may be attempted from any Section. All questions carry equal marks.

SECTION—A

- (i) A is a non-zero column and B, a non-zero row matrix, show that  $\rho(AB) = 1$ .  
(ii) Let A and B two square of order n, If  $\rho(A) = \rho(B) = n$  then show that  $\rho(AB) = n$  and conversely.
- Find the value of k so that the equations  $x - 2y + z = 0$ ,  $3x - y + 2z = 0$ ,  $y + kz = 0$  have :
  - Unique solution
  - Infinitely many solutions.

Also find solutions for these values of k.

### SECTION—B

3. (i) Prove that the characteristic roots of a Hermitian matrix are real.
- (ii) Prove that the characteristic roots of a skew Hermitian matrix  $A$  are either purely imaginary or zero.
4. Find the characteristic equation and Eigen values of the

$$\text{matrix } A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}.$$

### SECTION—C

5. (a) Every diagonal matrix whose diagonal elements are all positive is positive definite.
- (b) Every diagonal matrix whose diagonal elements are all negative is negative definite.
6. Express the quadratic form as a matrix product involving a symmetric coefficient matrix :
  - (a)  $Q = 8xy - x^2 - 31y^2$
  - (b)  $Q = 3x^2 - 2xy + 4xz + 5y^2 + 4z^2 - 2yz$

### SECTION—D

7. The roots of the equation  $x^3 - 12x^2 + 44x - 48 = 0$  are in A.P. Find them.
8. Solve the following equation by Descartes method  $x^4 - 6x^3 + 3x^2 + 22x - 6 = 0$